Spring 2017

Mathematical Sciences Colloquium

"A Roadmap to Well Posed and Stable Problems in Computational Physics"

All numerical calculations will fail to provide a reliable answer unless the continuous problem under consideration is well posed. Well-posedness depends in most cases only on the choice of boundary conditions. In this paper we will highlight this fact, and exemplify by discussing well-posedness of a prototype problem: the time-dependent compressible Navier–Stokes equations. We do not deal with discontinuous problems, smooth solutions with smooth and compatible data are considered.
In particular, we will discuss how many boundary conditions are required, where to impose them and which form they should have in order to obtain a well posed problem. Once the boundary conditions are known, one issue remains; they can be imposed weakly or strongly. It is shown that the weak and strong boundary procedures produce similar continuous energy estimates. We conclude by relating the well-posedness results to energy-stability of a numerical approximation on summation-by-parts form. It is shown that the results obtained for weak boundary conditions in the well-posedness analysis lead directly to corresponding stability results for the discrete problem, if schemes on summation-by-parts form and weak boundary conditions are used.
The analysis in this paper is general and can without difficulty be extended to any coupled system of partial differential equations posed as an initial boundary value problem coupled with a numerical method on summation-by parts form with weak boundary conditions. Our ambition in this paper is to give a general roadmap for how to construct a well posed continuous problem and a stable numerical approximation, not to give exact answers to specific problems.

Speaker: Jan Nordström

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Tuesday, March 7, 2017

Time: 4:00 – 5:00 PM

Location: Sage 3510

Host: Bill Henshaw

Rensselaer