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RENSSELAER POLYTECHNIC INSTITUTE

DEPARTMENT OF MATHEMATICAL SCIENCES COLLOQUIUM

Andrew Horning (MIT) December 11, 2023 - 4pm TROY 2012

Computational tools for the continuous spectrum of self-adjoint operators

Ever since Joseph Fourier used sines and cosines to diagonalize the Laplacian and solve the heat equation in 1822, spectral decompositions of linear operators have empowered scientists to disentangle complex physical phenomena. However, the spectrum of a self-adjoint operator can be more sophisticated than its familiar matrix counterpart; it may contain a continuum and the operator may not be diagonalized by eigenvectors alone. Until now, computational techniques for operators with continuous spectrum have typically focused on narrow classes with known analytical structure. Algorithms that achieve generality and rigor have been scarce.

In this talk, we present a tool kit of algorithms for computing spectral properties associated with the continuous spectrum of self-adjoint operators. These algorithms use the resolvent to construct high-order accurate approximations of inherently infinite-dimensional spectral properties, including smooth spectral measures, projections onto absolutely continuous subspaces, and non-normalizable modes. The algorithms are embarrassingly parallelizable and capable of leveraging state-of-the-art software for the resolvent of differential, integral, and lattice operators. Their flexibility and power are illustrated with applications in quantum and condensed matter physics and we highlight several exciting new developments in the growing class of resolvent-based algorithms for modal analysis in infinite-dimensional spaces.

Refreshments served at 3:30pm 4th floor Lounge – Amos Eaton

Biographical Sketch

Andrew Horning is an Applied Math Instructor at MIT Mathematics. His research interests are in numerical analysis and scientific computing with an emphasis on fast numerical linear algebra, approximation theory, and all things eigenvalue-related.

Horning completed his Ph.D. in applied mathematics at Cornell's Center for Applied Math, where he worked closely with his supervisor, Alex Townsend in the math department. Before that, he received a dual B.Sc in physics and mathematics from Rensselaer Polytechnic Institute (RPI), where he conducted research in cellular motors (math modeling), nanomaterials (computational physics), and ultrathin films (experimental physics). Horning's interests in math and physics accelerated after high school while working as an R&D technician at Praxis Technology, a small biomedical firm in Glens Falls, NY.



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